1. cvičení - výsledky

Příklad 1.

(a) $\frac{x^3+x^2+4x}{2(x-1)(x+1)^2\sqrt{x^3-1}}$

(b) $-\tan\left(e^x\right)\cdot e^x$

(c) $6\cdot\left(\cos(2x)-e^{2x}\right)^{-4}\cdot\left(\sin(2x)+e^{2x}\right)$

(d) $\frac{\log\left(x^3-2x+1\right)}{x\sqrt{x^2-1}}+\arccos\left(\frac{1}{x}\right)\cdot\frac{3x^2-2}{(x^3-2x+1)}$

Příklad 2.

(a) $f'_x(x,y)=6xy+35, [x,y]\in\mathbb{R}^2$
$f'_y(x,y)=8y+3x^2, [x,y]\in\mathbb{R}^2$

(b) $f'_x(x,y)=-\frac{\sin y^2}{x^2}, \{[x,y]\in\mathbb{R}^2\colon x\neq 0\}$
$f'_y(x,y)=\frac{2y\cos y^2}{x}, \{[x,y]\in\mathbb{R}^2\colon x\neq 0\}$

(c) $f'_x(x,y)=ye^{xy}+\cos(x+y), [x,y]\in\mathbb{R}^2$
$f'_y(x,y)=xe^{xy}+\cos(x+y), [x,y]\in\mathbb{R}^2$

(d) $f'_x(x,y)=\tan\frac{x}{y}+\frac{x}{y\cos^2\frac{x}{y}}, \{[x,y]\in\mathbb{R}^2\colon y\neq 0, \frac{x}{y}\neq\frac{\pi}{2}+k\pi, k\in\mathbb{Z}\}$
$f'_y(x,y)=\frac{-x^2}{y^2\cos^2\frac{x}{y}}, \{[x,y]\in\mathbb{R}^2\colon y\neq 0, \frac{x}{y}\neq\frac{\pi}{2}+k\pi, k\in\mathbb{Z}\}$

(e) $f'_x(x,y)=yx^{y-1}, \{[x,y]\in\mathbb{R}^2\colon x>0\}$
$f'_y(x,y)=x^y\log x, \{[x,y]\in\mathbb{R}^2\colon x>0\}$

(f) $f'_x(x,y)=\frac{x^2}{\sqrt[3]{(x^3+y^3)^2}}, \{[x,y]\in\mathbb{R}^2\colon y\neq -x\}$
$f'_y(x,y)=\frac{y^2}{\sqrt[3]{(x^3+y^3)^2}}, \{[x,y]\in\mathbb{R}^2\colon y\neq -x\}$

(g) $f'_x(x,y)=|y|\cdot\operatorname{sign}x, \{[x,y]\in\mathbb{R}^2\colon x\neq 0\}$
$f'_y(x,y)=|x|\cdot\operatorname{sign}y, \{[x,y]\in\mathbb{R}^2\colon y\neq 0\}$
Dále $fx'(0,0)=f'_y(0,0)=0$ a zbylé derivace neexistují.

(h) $f'_x(x,y)=\frac{2x}{3\sqrt[3]{(x^2+y^2)^2}}\log\left(x^2+y^2\right)+2x\frac{\sqrt[3]{x^2+y}}{x^2+y^2}, \{[x,y]\in\mathbb{R}^2\colon y\neq -x^2\}$
$f'_y(x,y)=\frac{1}{3\sqrt[3]{(x^2+y^2)^2}}\log\left(x^2+y^2\right)+2y\frac{\sqrt[3]{x^2+y}}{x^2+y^2}, \{[x,y]\in\mathbb{R}^2\colon y\neq -x^2\}$
Dále pokud $x^2+x^4\neq 1$, pak paricální derivace neexistují v bodě $(x,-x^2)$.
Pokud $x^2+x^4=1$, pak $f'_x(x,-x^2)=f'_y(x,-x^2)=0$

(i) $f'_x(x,y) = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{2x+y}{(x^2+xy+y^2)^2}, [x,y] \in \mathbb{R}^2 \setminus \{[0,0]\}$

$f'_y(x,y) = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{x+2y}{(x^2+xy+y^2)^2}, [x,y] \in \mathbb{R}^2 \setminus \{[0,0]\}$

A $f'_x(0,0) = f'_y(0,0) = 0$

Příklad 3.

(a) $f'_x(x,y,z) = yz + \log z, \{[x,y] \in \mathbb{R}^2 \colon z > 0\}$

$f'_y(x,y,z) = xz + z^2, \{[x,y] \in \mathbb{R}^2 \colon z > 0\}$

$f'_z(x,y,z) = xy + \frac{x}{z} + 2yz, \{[x,y] \in \mathbb{R}^2 \colon z > 0\}$

(b) $f'_x(x,y) = -\frac{y}{x^2-y^2}, \{[x,y] \in \mathbb{R}^2 \colon -x < y < x \vee x < y < -x\}$

$f'_y(x,y) = \frac{x}{x^2-y^2}, \{[x,y] \in \mathbb{R}^2 \colon -x < y < x \vee x < y < -x\}$

(c) $f'_x(x,y) = \frac{y}{x^2+y^2}, \{[x,y] \in \mathbb{R}^2 \colon x \neq y\}$

$f'_y(x,y) = -\frac{x}{x^2+y^2}, \{[x,y] \in \mathbb{R}^2 \colon x \neq y\}$

(d) $f'_x(x,y) = y \cdot \operatorname{sign} x, \{[x,y] \in \mathbb{R}^2 \colon x \neq 0\}$, dále $f'_x(0,0) = 0$ a f'_x neexistuje pro $x = 0, y \neq 0$

$f'_y(x,y) = |x|, [x,y] \in \mathbb{R}^2$

(e) $f'_x(x,y) = \frac{\sqrt[3]{y}}{3\sqrt[3]{x^2}}, \{[x,y] \in \mathbb{R}^2 \colon x \neq 0\}$

$f'_y(x,y) = \frac{\sqrt[3]{x}}{3\sqrt[3]{y^2}}, \{[x,y] \in \mathbb{R}^2 \colon y \neq 0\}$

Dále $f'_x(0,0) = f'_y(0,0) = 0$ a zbylé derivace neexistují.

(f) $f'_x(x,y) = -\operatorname{sign}(y - \sin x) \cdot \cos x, \{[x,y] \in \mathbb{R}^2 \colon y \neq \sin x\}$ a $f'_x(\frac{\pi}{2} + k\pi, (-1)^k) = 0, k \in \mathbb{Z}$

$f'_y(x,y) = \operatorname{sign}(y - \sin x), \{[x,y] \in \mathbb{R}^2 \colon y \neq \sin x\}$

(g) $f'_x(x,y,z) = \frac{z}{y}\left(\frac{x}{y}\right)^{z-1}, \{[x,y] \in \mathbb{R}^3 \colon xy > 0\}$

$f'_y(x,y,z) = -\frac{xz}{y^2}\left(\frac{x}{y}\right)^{z-1}, \{[x,y] \in \mathbb{R}^3 \colon xy > 0\}$

$f'_z(x,y,z) = \left(\frac{x}{y}\right)^{z} \log\frac{x}{y}, \{[x,y] \in \mathbb{R}^3 \colon xy > 0\}$

(h) $f'_x(x,y,z) = \frac{y}{z} \cdot x^{\frac{y}{z}-1}, \{[x,y] \in \mathbb{R}^3 \colon x > 0, z \neq 0\}$

$f'_y(x,y,z) = x^{\frac{y}{z}} \cdot \frac{\log x}{z}, \{[x,y] \in \mathbb{R}^3 \colon x > 0, z \neq 0\}$

$f'_z(x,y,z) = -x^{\frac{y}{z}} \cdot \frac{y \log x}{z^2}, \{[x,y] \in \mathbb{R}^3 \colon x > 0, z \neq 0\}$